C1/1 ALGEBRA

$a^m \cdot a^n = a^{m+n}$ $\qquad$ $a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{m \cdot n}$ $\qquad$ $a^{n/m} = (\sqrt[m]{a})^n$

$a^{-m} = 1/a^n$ $\qquad$ $a^1 = a$ $\qquad$ $a^0 = 1$

IMPORTANT!!!

$$ax^2 + bx + c = 0$$

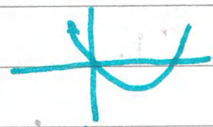
FACTORISING

QUADRATIC EQUATION

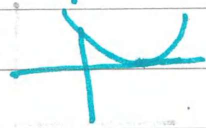
discriminant : $b^2 - 4ac$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where : $b^2 - 4ac > 0$
2 real roots

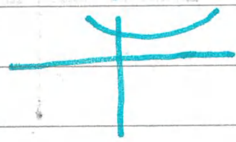


$b^2 - 4ac = 0$
1 real root



$y = c$ where $x = 0$

$b^2 - 4ac < 0$
no real roots



COMPLETING THE SQUARE

$$ax^2 + bx + c = 0$$

$$a[x^2 + b/a\, x + c/a = 0]$$

$$a[(x + b/2a)^2 - (b/2a)^2 + c/a = 0]$$

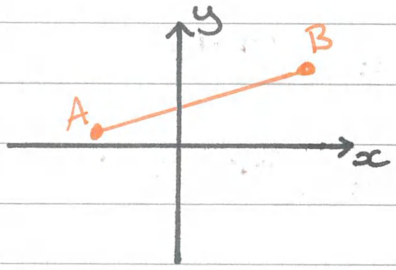
$$a(x + b/2a)^2 - a(b/2a)^2 + c = 0]$$

the form
$a(x+p)^2 + q$

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

$$\frac{a+b}{c+\sqrt{d}} = \frac{a+b}{c+\sqrt{d}} \times \frac{c-\sqrt{d}}{c-\sqrt{d}} = \frac{c(a+b) - \sqrt{d}(a+b)}{c^2 - d}$$

C1/2 COORDINATE GEOMETRY



GRADIENT $\overrightarrow{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

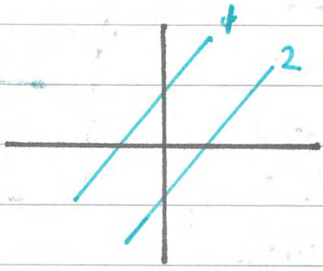
LENGTH $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

MIDPOINT $AB = \left(\dfrac{x_A + x_B}{2}, \dfrac{y_A + y_B}{2} \right)$

EQUATION OF A STRAIGHT LINE

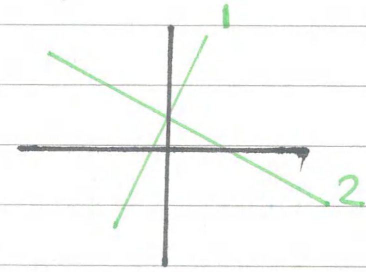
$y = mx + c$ $\quad$ m = gradient
c = y intercept $[x=0]$

OR $\quad y - y_1 = m(x - x_1)$ $\quad (x_1, y_1)$ point on line



PARALLEL LINES $\quad m_1 = m_2$
GRADIENTS ARE EQUAL

shorthand : llel



PERPENDICULAR LINES $\quad m_1 \times m_2 = -1$
INVERSE RECIPROCALS OF EACHOTHER

shorthand : $\perp^{r}$

C1/3 ALGEBRAIC MANIPULATION

FACTOR & REMAINDER THEOREM

$F(x) \,/\, (x-a)$

$F(x) = 0$: FACTOR

$F(a) = R$: REMAINDER

BINOMIAL THEOREM :

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$(a+b)^n = \left(a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \ldots + na^1b^{n-1} + b^n\right)$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

USE

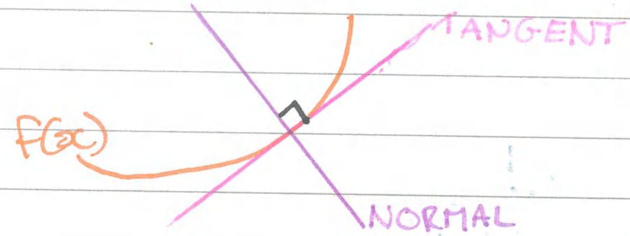
1 6 15 20 15 6 1 ← USE EQUATION ↓
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
...

C1/4 DIFFERENTIATION

FINDING THE GRADIENT ON A CURVE/TANGENT.

$$y = f(x) \qquad \frac{dy}{dx} = f'(x)$$

$$y = ax^n \qquad \frac{dy}{dx} = anx^{n-1}$$



$$m_{f(x)} = m_{TAN} = -1/m_{NORM}$$

↑ at point of intersection

STATIONARY POINTS $[dy/dx = 0]$

MAXIMUM POINT $d^2y/dx^2 < 0$

MINIMUM POINT $d^2y/dx^2 > 0$

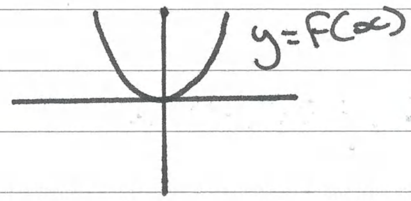
POINT OF INFLECTION $d^2y/dx^2 = 0$

$$\frac{d(dy/dx)}{dx} = \frac{d^2y}{dx^2}$$

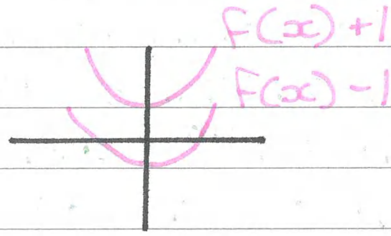
FIRST PRINCIPLES

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

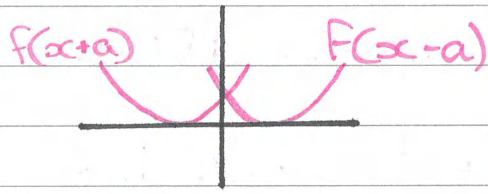
C1/5 TRANSFORMATIONS



$F(x) \pm a$ VERTICAL SLIDE ↑↓

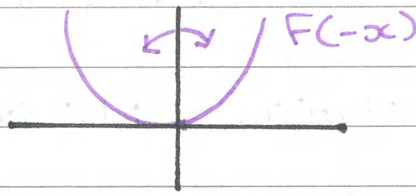


$F(x \pm a)$ NEG HORIZONTAL SLIDE ⇆

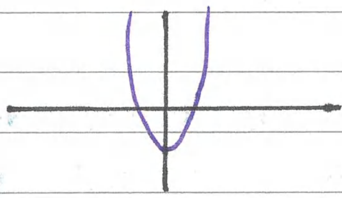


$-F(x)$ REFLECTION IN X AXIS

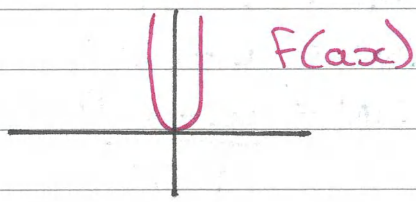
$F(-x)$ REFLECTION IN Y AXIS



$aF(x)$ STRETCH IN Y COORD



$F(ax)$ $1/a$ STRETCH IN X COORD



Kat Cole

C2/1 SEQUENCES & SERIES

$a =$ 1st TERM

$d =$ COMMON ~~RATIO~~ DIFFERENCE

$r =$ COMMON RATIO

$n =$ NUMBER OF TERMS

$U_N =$ Nth TERM

$S_N =$ SUM OF FIRST 'N' NUMBERS

ARITHMETIC PROGRESSIONS

$$U_N = a + (n-1)d$$

$$S_N = a + (a+d) + \dots + (a+(n-2)d) + (a+(n-1)d)$$

$$S_N = a+(n-1)d + a+(n-2)d + \dots + a+d + a$$

$$2S_N = 2a+(n-1)d + 2a+(n-1)d + \dots + 2a+(n-1)d + 2a+(n-1)d$$

$$2S_N = N[2a+(n-1)d]$$

$$S_N = \tfrac{N}{2}[2a+(N-1)d]$$

GEOMETRIC PROGRESSIONS

$$U_N = ar^{n-1}$$

$$S_N = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1}$$

$$rS_N = ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} + ar^n$$

$$S_N - rS_N = a - ar^n$$

$$(1-r)S_N = a - ar^n$$

$$S_N = \frac{a - ar^n}{1-r} \qquad S_N = \frac{a(1-r^n)}{1-r}$$

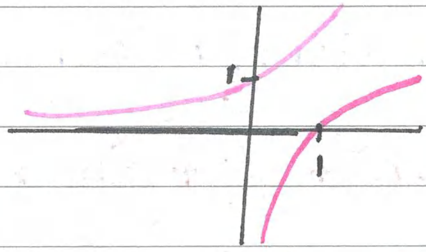
$$S_\infty = \frac{a}{1-r}$$

C2/2 EXPONENTIAL & LOGARITHMS

$$y = a^x \rightleftharpoons \log_a y = x$$

a = BASE $\qquad x$ = power

LOGARITHMIC GRAPH
$\ln(x)$
e^x
EXPONENTIAL GRAPH



LOG LAWS. IN ALL : let $a^b = x \Rightarrow \log_a x = b$
let $a^c = y \Rightarrow \log_a y = c$

$$xy = a^b \cdot a^c$$
$$xy = a^{b+c}$$
$$\log_a xy = b + c$$

$$[\log_a xy = \log_a x + \log_a y]$$

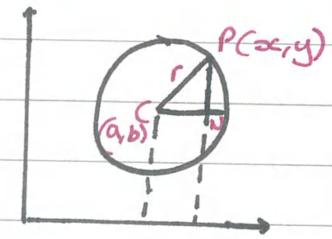
$$x/y = a^b/a^c$$
$$x/y = a^{b-c}$$
$$\log_a(x/y) = b - c$$

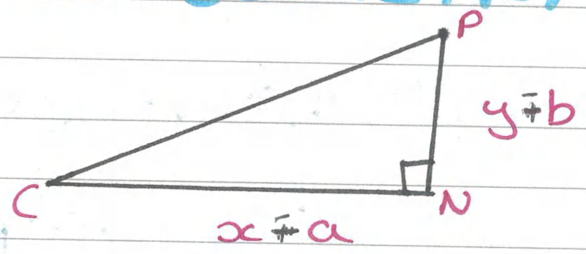
$$[\log_a(x/y) = \log_a x - \log_a y]$$

$$(a^b)^n = x^n$$
$$a^{b.n} = x^n$$
$$\log_a x^n = bn$$
$$\log_a x^n = n\log_a x$$

$$[\log_a x^n = n \log_a x]$$

C2/3 COORDINATE GEOMETRY



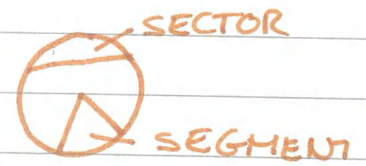


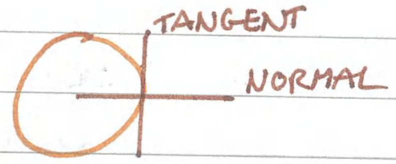
CENTRE RADIUS FORM : $(x \mp a)^2 + (y \mp b)^2 = r^2$

$$x^2 + y^2 + ax + by + \overset{3}{c} = 0$$

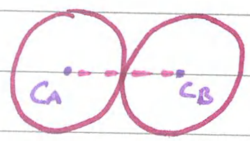
$$\underbrace{\left(x+\tfrac{a}{2}\right)^2 - \left(\tfrac{a}{2}\right)^2}_{1} + \underbrace{\left(y+\tfrac{b}{2}\right)^2 - \left(\tfrac{b}{2}\right)^2}_{2} + \underbrace{c}_{3} = 0$$

$$\left(x+\tfrac{a}{2}\right)^2 + \left(y+\tfrac{b}{2}\right)^2 = \left(\tfrac{a}{2}\right)^2 + \left(\tfrac{b}{2}\right)^2 - c$$





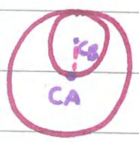
CIRCLES THAT TOUCH EXTERNALLY



$C_A C_B = r_A + r_B$

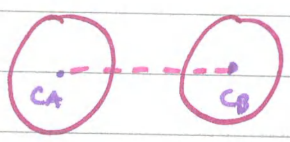
LENGTH OF

CIRCLES THAT TOUCH INTERNALLY

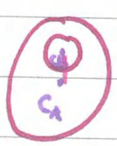


$C_A C_B = r_A - r_B$

CIRCLES THAT DONT TOUCH EXTERNALLY / INTERNALLY

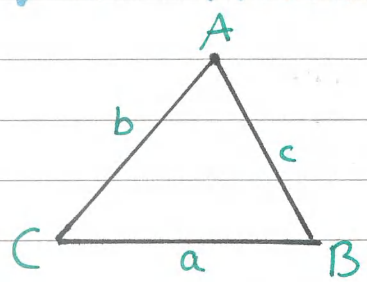


$C_A C_B > r_A + r_B$



$C_A C_B < r_A + r_B$

C2/4 TRIGONOMETRY



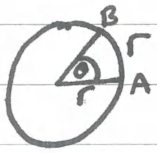
SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

COSINE RULE $a^2 = b^2 + c^2 - 2bc\cos A$

AREA OF TRIANGE $= \frac{1}{2}ab\sin C$

RADIANS & DEGREES



$1^c \simeq 57.3^\circ$

AREA SECTOR $= \frac{1}{2}\theta r^2$

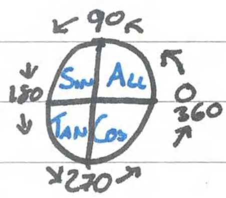
Arc AB $= \theta r$

$\theta \times \frac{\pi}{180} = c$

$c \times \frac{180}{\pi} = \theta$

θ = DEGREES

c = RADIANS

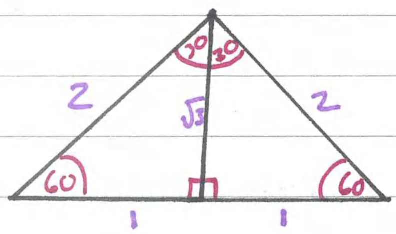


CAST METHOD

shows where θ is pos.

IDENTITIES $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$

$\sin^2\theta + \cos^2\theta \equiv 1$



$\sin 30 = \frac{1}{2}$

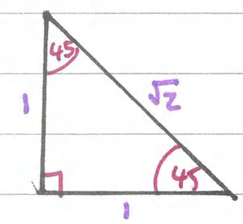
$\cos 30 = \frac{\sqrt{3}}{2}$

$\tan 30 = \frac{1}{\sqrt{3}}$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{1}{2}$

$\tan 60 = \sqrt{3}$



$\sin 45 = \frac{1}{\sqrt{2}}$

$\cos 45 = \frac{1}{\sqrt{2}}$

$\tan 45 = 1$

$2\pi^c = 360^\circ$

$\pi^c = 180^\circ$

$\frac{\pi}{2}^c = 90^\circ$

$\frac{\pi}{4}^c = 45^\circ$

$\frac{\pi}{3} = 60^\circ$

$\frac{\pi}{6} = 30^\circ$

C2/5 INTEGRATION

$$\frac{dy}{dx} = ax^n \qquad y = \frac{ax^{n+1}}{n+1}$$

$$\int f'(x)\,\delta x = f(x) + c$$

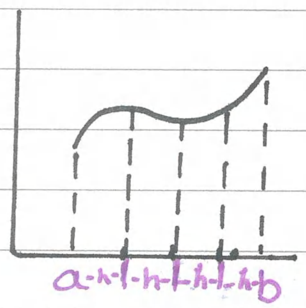
$$\int ax^n\,\delta x = \frac{ax^{n+1}}{n+1} + c$$

$$\int a\,\delta x = ax + c$$

AREA UNDER A CURVE

$$\lim_{\delta x \to 0} \sum_i y\,\delta x = \int_a^b y\,\delta x$$

THE TRAPEZIUM RULE



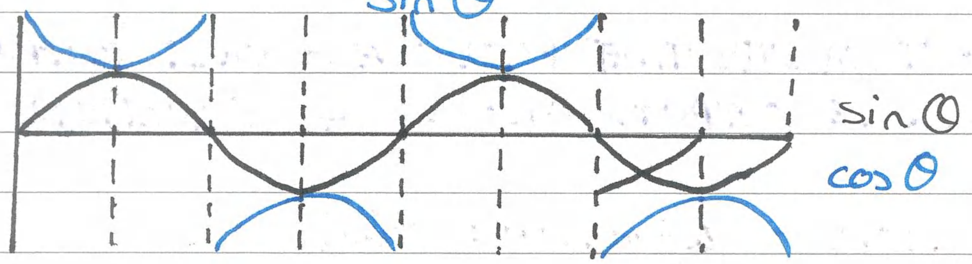
AREA UNDER CURVE DEFINED BY:

$$A = \sum_i \left(\frac{a+b}{2} h\right) \quad \text{where } h = \frac{b-a}{n}$$

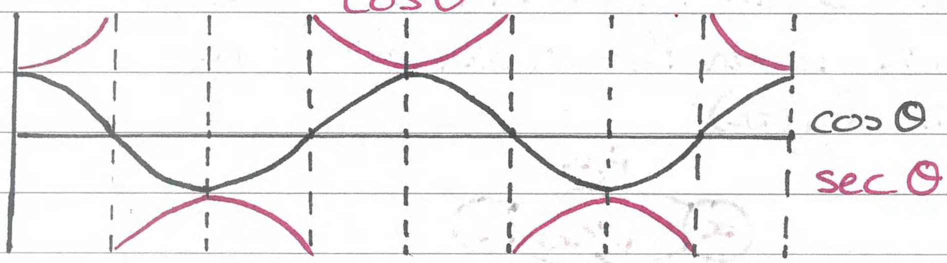
$$A = \frac{h}{2}\left[\,y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

C3/1 TRIGONOMETRY

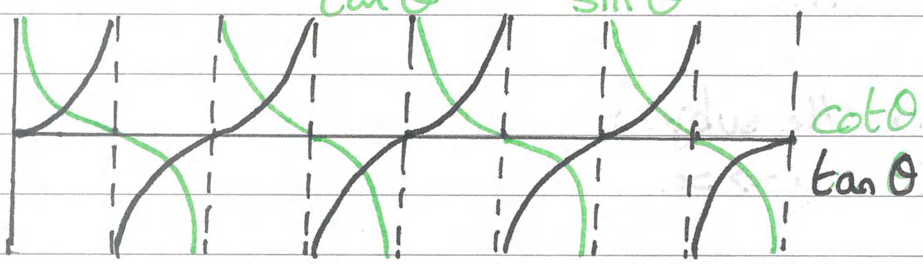
$\operatorname{Cosec}\theta = \dfrac{1}{\sin\theta}$



$\operatorname{Sec}\theta = \dfrac{1}{\cos\theta}$



$\operatorname{Cot}\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$



IDENTITIES

$\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$

$\sec^2\theta \equiv 1 + \tan^2\theta$

DONT FORGET:

$\sin^2\theta + \cos^2\theta \equiv 1 \qquad \dfrac{\sin\theta}{\cos\theta} \equiv \tan\theta$

C3/2 FUNCTIONS

FUNCTION : MAPPING WITH ONE-ONE OR MANY TO ONE

DOMAIN : ALL INPUT VALUES THAT PRODUCE A VALID OUTPUT
RANGE : ALL POSSIBLE OUTPUTS FROM A FUNCTION

COMPOSITE FUNCTIONS 1 FUNCTION : $f(x)$
$fg(x) = f \circ g(x) = f(g(x))$ f of g of x $f^2(x) = f(f(x))$

$fg(x)$ x MUST be in the domain of $g(x)$
$g(x)$ MUST be in the domain of $f(x)$
OUTPUT of that is $fg(x)$

INVERSE FUNCTION

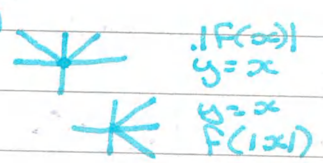
$$\begin{tikzcd} x \arrow[r, bend left, "f(x)"] & y \arrow[l, bend left, "f^{-1}(x)"] \end{tikzcd}$$

DOMAIN OF $f(x) \equiv$ RANGE OF $f^{-1}(x)$
RANGE OF $f(x) \equiv$ DOMAIN OF $f^{-1}(x)$
- To find formula of $f^{-1}(x)$
- Swap $f(x)$ with y
- rearrange to make x the subject
- replace $x \Rightarrow f^{-1}(x)$ $y \Rightarrow x$

MODULUS

$|x| < n$ means $-n < x < n$
$x < n$
$-x < n$ $x > -n$

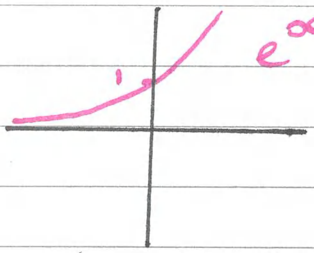
$|f(x)| = f(x)$ when $f(x) \geq 0$
$f(|x|)$ $= -f(x)$ when $f(x) < 0$

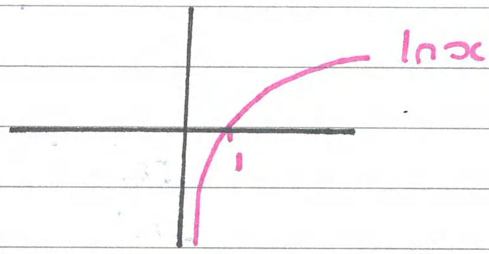


IF $|a| = |b|$ then $a^2 = b^2$

$y = |f(x)|$ whole graph pos in y ∴ reflected in x axis
$y = f(|x|)$ all x values are pos ∴ reflect in y axis

C3/3 LOG AND EXPONENTIALS





$\ln x$ is the INVERSE of e^x

$y = e^x$ $\quad \log_e y = x$ $\quad \log_e = \ln$

EXPONENTIAL FUNCTION $\Rightarrow$ where x is in the power: a^x
The x axis is an asymptote to all a^x for any a

$y = e^x$ $\quad \frac{dy}{dx} = e^x$ $\quad$ it is its own gradient function

$e^1 \simeq 2.718281828$

$\ln(e^x) = x$ $\quad e^{\ln(x)} = x$

$\ln(e) = 1$ $\quad \ln(1) = 0$

when solving equations:

IF $\ln(x)$ take e both sides
$\ln(x) = a \Rightarrow x = e^a$

IF e^x take $\ln$ both sides
$e^x = a$ $\quad x = \ln(a)$

C3/4 DIFFERENTIATION

y	dy/dx
c	0
x	1
ax^n	anx^{n-1}
$K\sin(ax+b)$	$aK\cos(ax+b)$
$K\cos(ax+b)$	$-aK\sin(ax+b)$
$K\tan(ax+b)$	$aK\sec^2(ax+b)$
e^{Kx}	Ke^{Kx}
$\ln(ax+b)$	$a/ax+b$
$\sin^{-1}(x)$	$1/\sqrt{1-x^2}$
$\cos$ ~~sin~~$^{-1}(x)$	$-1/\sqrt{1-x^2}$
$\tan^{-1}(x)$	$1/x^2+1$

CHAIN RULE

$y = F(u) \qquad u = f(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

PRODUCT RULE

$y = u(x) . v(x)$

$$\frac{dy}{dx} = u.v' + v.u'$$

QUOTIENT RULE

$y = u/v$

$$\frac{dy}{dx} = \frac{v.u' - u.v'}{v^2}$$

PARAMETRIC EQUATIONS

$x = f(t) \qquad y = g(t)$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

IMPLICIT DIFFERENTIATION

- DIFFERENTIATE $f(y)$ WITH RESPECT TO x

$$\frac{d(f(y))}{dx} \times \frac{dy}{dx}$$

Solve for $\frac{dy}{dx} = n$

C3/5 INTEGRATION

y	$\int y \, \delta x$
1	$x + c$
a	$ax + c$
$x^n \ (n \neq 1)$	$\frac{x^{n+1}}{n+1} + c$
$1/x$	$\ln x + c$
$e^{(ax+b)}$	$\frac{e^{ax+b}}{a} + c$
$K\cos(ax+b)$	$\frac{K\sin(ax+b)}{a} + c$
$K\sin(ax+b)$	$\frac{-K\cos(ax+b)}{a} + c$
$K/ax+b$	$\frac{K.\ln\lvert ax+b \rvert}{a} + c$
$K(ax+b)^n$	$\frac{K(ax+b)^{n+1}}{a(n+1)} + c$

DEFINITE INTEGRALS - TRAPEZIUM RULE
- AREA UNDER A CURVE

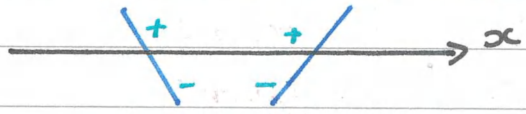
$$\int_a^b y \, \delta x = \left[y \delta x \right]_a^b$$

$$= \left[b \delta x \right] - \left[a \delta x \right]$$

where $b > a$

C3/6 NUMERICAL METHODS

ROOT : WHERE A FUNCTION CROSSES THE X AXIS



IF THERE IS A ROOT BETWEEN TWO VALUES THERE WILL BE A CHANGE IN SIGN

ITERATIVE METHODS - A RECCURANCE RELATION
- helps to get closer to the approximate value of the root
• INPUT VALUE x_0 INTO ITERATIVE FORMULA UNTIL n DECIMAL PLACES ARE CONSTANT.

$$x_{n+1} = \frac{a}{F(x_n)}$$

SIMPSONS RULE
- more accurate approximation than the trapezium rule because the tops of trapezium are approximated to quadratic curves

$$\int_a^b F(x)\,\delta x \approx \frac{h}{3}\left[y_0 + y_n + 4(y_1 + \dots + y_{n-1}) + 2(y_2 + \dots + y_{n-2})\right]$$

$\frac{b-a}{n} = h$ = WIDTH OF TRAPEZIUM

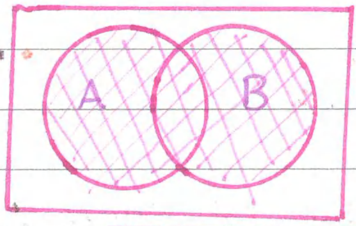
n = number of trapezia
Ordinates = n+1

S1/1 PROBABILITY

COMBINED EVENTS

$P(A \cup B)$

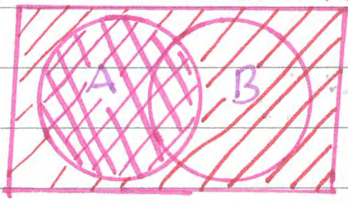
A OR B OR BOTH



$P(A) + P(B) - P(A \cap B)$

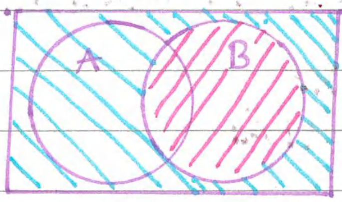
$P(A)$

$P(A') = \cancel{P} \; 1 - P(A)$



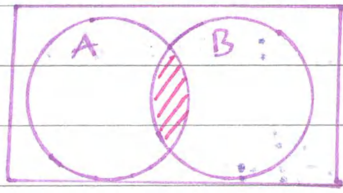
$P(B)$

$P(B') = 1 - P(B)$



$P(A \cap B)$

A AND B



$P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ $\qquad P(A/B).P(B) = P(B/A).P(A)$

A given B

INDEPENDANT EVENTS - no effect on each other

$P(A/B) = P(A)$

$P(A \cap B) = P(A) \times P(B)$

MUTUALLY ~~EXHAUSTIVE~~ EXCLUSIVE (A OR B) CANT BE BOTH

$P(A \cup B) = P(A) + P(B)$

$P(A \cap B) = 0$

$P(A) = P(A \cap B) + P(A \cap B')$

EXHAUSTIVE EVENTS

$P(A \cup B) = 1$

S1/2 PERMUTATIONS & COMBINATIONS

ARRANGING X NUMBER OF OBJECTS IN A LINE
(number of ways of)

$x!$ $\quad {}^{x}P_{x}$ $\quad x=6$ $\quad 6!$

ARRANGING X NUMBER OF OBJECTS OF WHICH Y MANY ARE ALIKE

$x!/y!$ $\quad x=6$ $\quad y=3$ $\quad 6!/3!$

ARRANGING X NUMBER OF WHICH DIFFERENT SETS ARE ALIKE

$$\frac{x!}{y!\,z!\,A!} \qquad \frac{11!}{2!\,3!\,4!} \qquad x=11,\ y=2,\ z=3,\ A=4$$

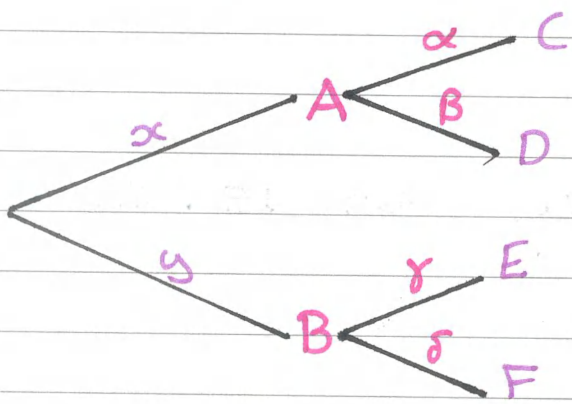
COMBINATIONS OF Y FROM X DIFFERENT OBJECTS

$${}^{x}C_{y} \qquad \frac{x!}{(n-r)!\,r!}$$

PERMUTATIONS OF Y FROM X UNLIKE

$${}^{x}P_{y} \qquad \frac{n!}{(n-r)!}$$

TREE DIAGRAMS



$x+y=1$

$\alpha+\beta=1$

$\gamma+\delta=1$

S1/3 Discrete Probability Distribution

x^2	a^2	
$1/x$	$1/a$	
x	a	....
$P(X=x)$	b	

$$\sum P(X=x) = 1$$

$E(X)$ = expected value of x = $\sum_{\text{ALL}} x . P(X=x) = \sum ab$

$E(X^2)$ = expected value of x^2 = $\sum_{\text{ALL}} x^2 . P(X=x) = \sum a^2 b$

$E(1/x)$ = expected value of $1/x$ = $\sum_{\text{ALL}} \frac{1}{x} . P(X=x) = \sum b/a$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(aX+b) = aE(X) + b$$

$$Var(aX+b) = a^2 Var(X)$$

S1/4 Binomial & Poisson Distributions

Binomial Distribution

- Constant probability of success/failure
- Fixed number of trials
- Only two options - success/failure

$X \sim B(n,p)$ n = n.o. trials p = success q = failure

$$P(X=x) = {}^nC_x \cdot p^x \cdot q^{1-x}$$

$E(x) = np$ $Var(x) = npq$

Poisson Distribution

- Events occur singularly & at random
- N.O. occurances is known & finite - λ

$$X \sim Po(\lambda)$$

$$P(X=x) = e^{-\lambda}\left(\frac{\lambda^x}{x!}\right)$$

$E(x) = \lambda$ $Var(X) = \lambda$

Poisson Approximation

- $n > 50$
& $p < 0.1$

$X \sim Bin(n,p)$ $n \cdot p = \lambda$

$X \sim Po(\lambda)$

S1/5 CONTINUOUS PROBABILITY

$$\int ax^n \,\delta x = \frac{ax^{n+1}}{n+1}$$

$$\int_{ALL} F(x)\,\delta x = 1$$

$$E(X) = \mu = \int_{all} x.f(x)\,\delta x$$

$$E(X^2) = \mu^2 = \int_{all} x^2.F(x)\,\delta x$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(aX+b) = aE(X)+b$$

$$Var(aX+b) = a^2 Var(X)$$

$$f(x) \rightleftharpoons F(x)$$ INTEGRATE / DIFFERENTIATE

IF $a \leqslant x \leqslant b$

$F(a) = 0$ $F(b) = 0$

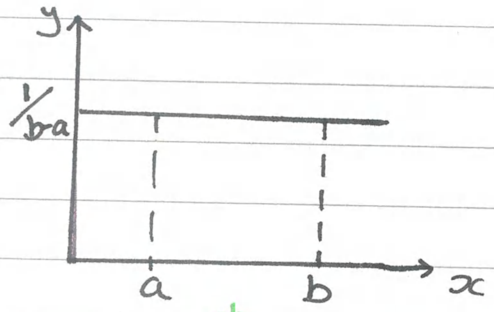
$F(\text{median}) = 0.5$

$F(\text{Lower Quartile}) = 0.25$

$F(\text{Upper Quartile}) = 0.75$

$F(IQR) = F(UQ) - F(LQ)$

S2/1 UNIFORM DISTRIBUTION



$a < x < b$

$F(x) = \frac{1}{b-a}$

$X \sim U(a,b) \qquad X \sim R(a,b)$

$$E(x) = \int_a^b x \cdot \frac{1}{b-a}\, \delta x$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{b^2-a^2}{2(b-a)} = \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}} = \boxed{\frac{b+a}{2}}$$

$$E(x^2) = \int_a^b x^2 \cdot \frac{1}{b-a}\, \delta x$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{b^3-a^3}{3(b-a)} = \frac{\cancel{(b-a)}(a^2+ab+b^2)}{3\cancel{(b-a)}} = \boxed{\frac{a^2+ab+b^2}{3}}$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{(b+a)^2}{4}$$

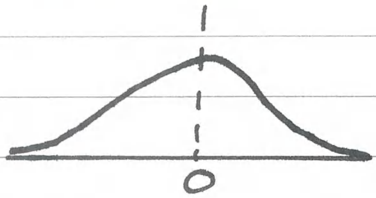
$$= \frac{4(a^2+ab+b^2) - 3(b+a)^2}{12}$$

$$= \frac{4a^2 + 4ab + 4b^2 - 3b^2 - 6ba - 3a^2}{12}$$

$$= \boxed{\frac{a^2-2ab+b^2}{12}}$$

$$Var(x) = \boxed{\frac{(b-a)^2}{12}} \qquad E(x) = \boxed{\frac{b+a}{2}}$$

S2/2 NORMAL DISTRIBUTION



IF $Z \sim N(0,1)$ USE TABLE

mean ↑ st. dev ↑

IF $X \sim N(\mu,\sigma^2)$, $Z \sim N\left(\frac{X-\mu}{\sigma^2}, \sqrt{\sigma^2}\right)$

then $Z = \frac{X-\mu}{\sigma^2}$ $\Rightarrow$ standardising formula

$P(X>n) \Rightarrow P(Z>n)$ USE TABLE

$\Phi = P(Z<n)$ $\Phi(n) = P(Z<n)$ MODULUS: negatives are irrelevant

CONTINUITY CORRECTIONS

$P(X \geq n) = P(X > n-0.5)$

$P(X > n) = P(X > n+0.5)$

$P(X \leq n) = P(X < n+0.5)$

$P(X < n) = P(X < n-0.5)$

$X \sim N(\mu, \sigma^2)$

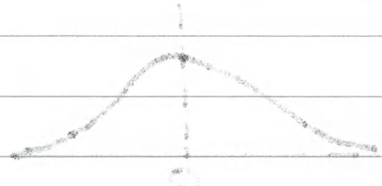
$X \sim Bin(n,p) \Rightarrow X \sim N(np, npq)$

when $p \approx \frac{1}{2}$ $n = $ v. large

$X \sim Po(\lambda) \Rightarrow X \sim N(\lambda, \lambda)$

↳ $\lambda > 15$

S2/3 LINEAR COMBINATIONS OF VARIABLES



$E(X+Y) = E(X) + E(Y)$

$E(X-Y) = E(X) - E(Y)$

$E(aX+bY) = aE(X) + aE(Y)$

$E(XY) \doteq E(X).E(Y)$

$Var(X+Y) = Var(X) + Var(Y)$

$Var(X-Y) = Var(X) + Var(Y)$

$Var(aX+bY) = a^2 Var(X) + b^2 Var(Y)$

$X \sim N(\mu_1, \sigma_1^2) \qquad Y \sim N(\mu_2, \sigma_2^2)$

$X+Y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

$X-Y \sim N(\mu_1-\mu_2, \sigma_1^2+\sigma_2^2)$

$X_1 \sim N(\mu_1, \sigma_1^2) \; \ldots \; X_n \dot{\sim} N(\mu_n, \sigma_n^2)$

$X_1 + X_2 + \ldots X_n \sim N(\mu_1+\mu_2+\ldots \mu_n, \sigma_1^2+\sigma_2^2+\ldots\sigma_n^2)$

$X \sim N(\mu, \sigma^2)$

$aX \sim N(a\mu, a^2\sigma^2)$

$X \sim N(\mu_x, \sigma_x^2) \qquad Y \sim N(\mu_y, \sigma_y^2$

$aX+bY \sim N(a\mu_x + b\mu_y, a^2\sigma^2_x + b^2\sigma^2_y)$

$aX-bY \sim N(a\mu_x - b\mu_y, a^2\sigma^2_x + b^2\sigma^2_y)$

$X \sim Po(m) \qquad Y \sim Po(n)$

$X+Y \sim Po(m+n)$

$E(X_1+X_2+\ldots+X_n) = E(X_1) + E(X_2) + \ldots E(X_n)$

$Var(X_1+X_2+\ldots X_n) = 1^2 Var(X_1) + \ldots \ldots 1^2 Var(X_n)$

$= n Var(X)$

$E(X_1 - X_2) = E(X) - E(X) = 0$

$Var(X_1 - X_2) = Var(X) + Var(X) = 2Var(X)$

S2/4 DISTRIBUTION OF MEAN

$X \sim N(\mu, \sigma^2) \implies \bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ n = size of random sample

POPULATION SAMPLE

STANDARDISE ERROR OF MEAN: STANDARDISE DEVIATION OF $\bar{X}$

$= \sqrt{\frac{\sigma^2}{n}}$

$= \frac{\sigma}{\sqrt{n}}$

CENTRAL LIMIT THEOREM

all Distributions become normal with large n.

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

$$X \sim Po(\lambda) \implies \bar{X} \sim N(\lambda, \lambda/n)$$

$$X \sim Bin(n,p) \implies \bar{X} \sim N\left(np, \frac{npq}{n}\right)$$

$$X \sim R(a,b) \implies \bar{X} \sim N\left(\frac{a+b}{2}, \frac{(b-a)^2}{12n}\right)$$

S2/5 HYPOTHESIS TESTING

H_0 - NULL HYPOTHESIS
H_1 - ALTERNATIVE HYPOTHESIS
TEST STAT - TEST VALUE FROM DATA
SIGNIFICANCE LEVEL - P(REJECTING H_0)
CRITICAL REGION - VALUES WITH WHICH REJECT H_0
P VALUE - P(CURRENT DATA ASSUMING H_0 IS TRUE)

$p < 0.01 \Rightarrow$ V STRONG EVIDENCE
$0.01 \leq p \leq 0.05 \Rightarrow$ STRONG EVIDENCE
$p > 0.05 \Rightarrow$ INSUFFICIENT EVIDENCE

ACCEPT/REJECT H_0 ← DEPENDING ON WHERE TEST STAT FALLS

METHOD
$H_0 = x \quad H_1 = y$
DEFINED INCREASE/DECREASE = ONE TAILED. IF NOT = TWO TAILED
UNDER H_0 CONSIDER APPROPRIATE DISTRIBUTION
USE p VALUES OR CRITICAL REGION
ACCEPT/REJECT $H_0 \Rightarrow$ DRAW CONCLUSION

DIFFERENCE BETWEEN TWO MEANS

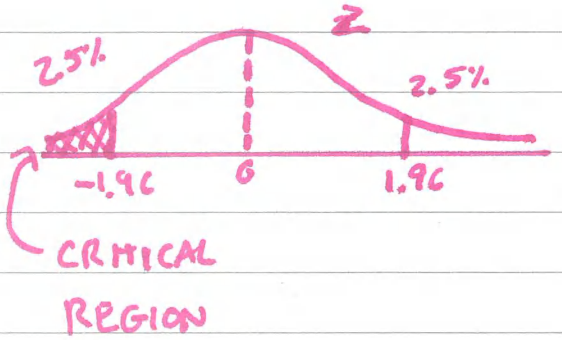
$\bar{X}_1 \sim N(\mu_1, \sigma_1^2/n_1) \qquad \bar{X}_2 \sim N(\mu_2, \sigma_2^2/n_2)$

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

UNDER H_0 : $\mu_1 = \mu_2 \qquad \mu_1 - \mu_2 = 0$

TEST STAT : $z = \dfrac{x - \mu}{\sigma}$

5% SIG LEVEL



EVIDENCE TO REJECT H_0 ≠ IN C.R.
ACCEPT H_0 IN C.R.

S2/6 CONFIDENCE INTERVALS

σ^2 MUST BE ESTIMATED $= \frac{\sum x^2}{n} - \bar{x}^2$

UnBiased Estimate $\Rightarrow UBE(\sigma^2) = \frac{n(\sigma^2)}{n-1} = s^2$

CONFIDENCE LIMITS

$\left[\bar{X} - Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right), \bar{X} + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right)\right]$ Z_α - point on t-distribution

$X \sim N(\mu, s^2)$ $\bar{X} \sim N(\mu, s^2/n)$

$\nu = n-1$ ∴ degree of freedom → t-distribution

Binomial Probability $\theta = P(\text{success})$

$UE(\theta)$ $p = x/n$ $ESE(p) = \sqrt{\frac{p(1-p)}{n}}$

$[p \pm Z_\alpha ESE(p)]$ $p_s \sim N\left(\theta, \frac{\theta(1-\theta)}{n}\right)$

Mean (of any dist) σ^2 unknown $\Rightarrow UBE(\sigma^2) = s^2$

$s^2 = \frac{n\sigma^2}{n-1}$

DIFFERENCE BETWEEN TWO MEANS

$\bar{X} - \bar{Y} \sim N\left(\mu_x - \mu_y, \frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)$

$ESE(\bar{X} - \bar{Y}) = \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$ $C.L. = \mu_x - \mu_y$

$\bar{x} - \bar{y} \pm Z_\alpha \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$

S3/1 SAMPLING DISTRIBUTION

WITHOUT REPLACEMENT

POSSIBLE SAMPLES	PROBABILITY	MEAN	MEDIAN

SAMPLING DIST MEAN $E(\bar{x}) = E(x) = \mu$

SAMPLING DIST MEDIAN $E(\bar{m}) \neq E(m)$

USE COMBINATIONS OR FRACTIONS FOR PROB

WITH REPLACEMENT - SAMPLING DIST OF MEAN

$X \sim Bin(n, p)$ X = n.o. successes $P(x) = P_s$

$P_s = \frac{X}{n}$ WHEN n IS LARGE $P_s \sim N(p, \frac{pq}{n})$

$E(P_s) = E(\frac{X}{n}) = \frac{1}{n} E(X) = \frac{np}{n} = p$

$Var(P_s) = Var(X/n) = \frac{1}{n^2} Var(X) = \frac{npq}{n^2} = \frac{pq}{n}$

std. dev $= \sqrt{\frac{pq}{n}}$

UNBIASED ESTIMATORS (UBE)

$\sigma^2 = \frac{n\sigma^2}{n-1}$ (S) $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$

EFFICIENT ESTIMATOR $= \frac{\sum x^2 - n\bar{x}^2}{n-1}$

S3/2 ESTIMATORS

SAMPLE STATISTIC CHOSEN TO ESTIMATE A POPULATION PARAMETER
ESTIMATOR SHOULD BE UNBIASED
- SMALLER ESTIMATOR VARIENCE = BETTER
UBE : $E(x) = \mu$
TO FIND VARIABLE/MINIMISE VAR(x) : $\frac{d(\mathrm{Var}(x))}{dn} = 0$

UNBIASED ESTIMATORS
- POPULATION MEAN

$E(\bar{x}) = \mu$ $\quad \mathrm{Var}(\bar{x}) = \frac{\sigma^2}{n}$ $\quad SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$ $\quad$ var $\downarrow$ as $n \uparrow$

BINOMIAL PROBABILITY $X \sim \mathrm{Bin}(n, \theta)$
θ is unknown

$p = \frac{X}{n}$ $\quad E(p) = \theta$ $\quad \mathrm{Var}(p) = \frac{\theta(1-\theta)}{n}$

$SE(p) = \sqrt{\frac{\theta(1-\theta)}{n}}$ $\quad ESE(p) = \sqrt{\frac{p(1-p)}{n}}$

POPULATION VARIENCE

$S^2 = \frac{1}{n-1}(\Sigma x^2 - n\bar{x}^2)$ $\quad S^2 = \frac{ns^2}{n-1}$

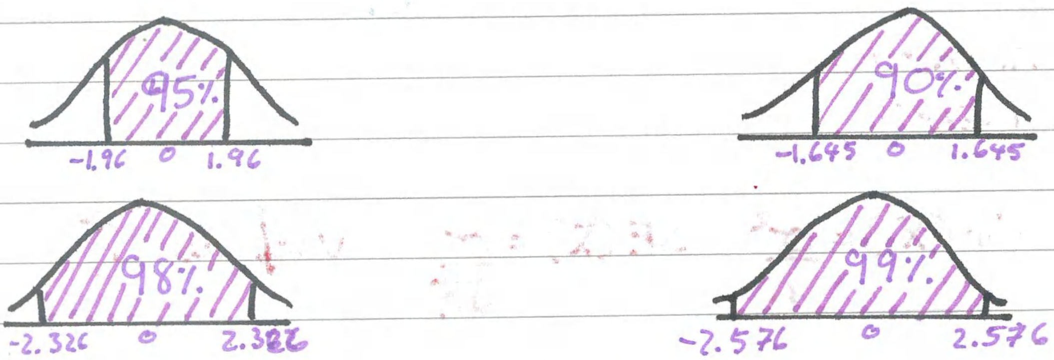
$\mathrm{Var}(\bar{x}) = \frac{s^2}{n}$ $\quad ESE(\bar{x}) = \frac{s}{\sqrt{n}}$

S3/3 CONFIDENCE INTERVALS

2 TAILED

$X \sim N(\mu, \sigma^2)$ $\qquad$ $\bar{X} \sim N(\mu, \sigma^2/n)$

$$\left(\bar{X} - z\sqrt{\frac{\sigma^2}{n}}\,,\ \bar{X} + z\sqrt{\frac{\sigma^2}{n}}\right)$$



$X_1 \sim N(\mu_1, \sigma_1^2)$ $\qquad$ $\bar{X}_1 \sim N(\mu_1, \sigma_1^2/n_1)$

$X_2 \sim N(\mu_2, \sigma_2^2)$ $\qquad$ $\bar{X}_2 \sim N(\mu_2, \sigma_2^2/n_2)$

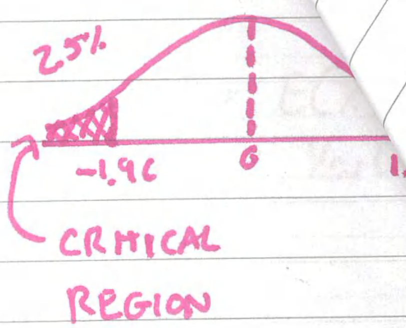
$$\left(\bar{x}_1 - \bar{x}_2 - z\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}\,,\ \bar{x}_1 - \bar{x}_2 + z\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}\right)$$

$\bar{X}_1$ … $\bar{x}_1$ AND $\bar{x}_2$ ARE SAMPLE MEANS OF X_1 & X_2

UNDER …

TEST ST…

5% SIG L…

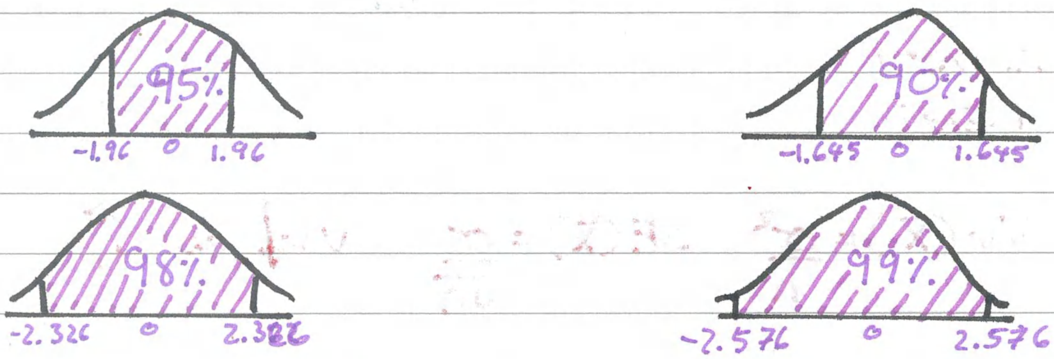


S3/3 CONFIDENCE INTERVALS

2 TAILED

$X \sim N(\mu, \sigma^2)$ $\qquad$ $\bar{X} \sim N(\mu, \sigma^2/n)$

$$\left(\bar{X} - Z\sqrt{\frac{\sigma^2}{n}}\ ,\ \bar{X} + Z\sqrt{\frac{\sigma^2}{n}}\right)$$



$X_1 \sim N(\mu_1, \sigma_1^2)$ $\qquad$ $\bar{X}_1 \sim N(\mu_1, \frac{\sigma_1^2}{n_1})$

$X_2 \sim N(\mu_2, \sigma_2^2)$ $\qquad$ $\bar{X}_2 \sim N(\mu_2, \sigma_2^2/n_2)$

$$\left(\bar{x}_1 - \bar{x}_2 - Z\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}\ ,\ \bar{x}_1 - \bar{x}_2 + Z\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}\right)$$

WHERE $\bar{x}_1$ AND $\bar{x}_2$ ARE SAMPLE MEANS OF X_1 & X_2

S3/4
HYPOTHESIS TESTING

STANDARD DEVIATION IS GIVEN BY

$$\sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$UBE(s) = \frac{ns^2}{n-1}$$

C.L.T : $\bar{X} \sim N(\mu, \sigma^2/n)$

S3/5 LINEAR RELATIONSHIPS

$$y = a + bx + \varepsilon$$

ε = error $\qquad \varepsilon \sim N(0, \sigma^2)$

FIND: $\bar{x}, \bar{y}, n, \Sigma x, \Sigma y, \Sigma x^2, \Sigma yx$

$$S_{xy} = \Sigma xy - \frac{\Sigma x . \Sigma y}{n}$$

$$S_{xx} \ \Sigma x^2 - \frac{[\Sigma x]^2}{n}.$$

$$b = \frac{S_{xy}}{S_{xx}} \qquad a = \bar{y} - b\bar{x}$$

$$a \sim N\left(\alpha, \frac{\sigma^2 \Sigma x^2}{n S_{xx}}\right)$$

$$b \sim N\left(\beta, \frac{\sigma^2}{S_{xx}}\right)$$

$$y = a + bx \sim N\left(y, \sigma^2\left[\frac{1}{n} + \frac{(x - \bar{x})^2}{S_{xx}}\right]\right)$$